Is 4/n is always the sum of three Egyptian fractions, for n>1

Reference: [Erdos 1980]

Paul erdos and Ronald L.Graham, old and new problems and results in combination number theory.

L'Enseignment mathematique Geneva, Switzerland: 1980, Page 44.

Abstract: A number of the form 1/x where x is an integer is called an egyption fraction.

The mathematical equivalence would be whether.

$4/x = 1/x+1/y+1/z$ $\quad\quad\quad\quad$ $n>1$

The result has been established for all even numbers, all odd numbers of the form 4x-1. All odd w.s of the form 24n+1 are only undecidabale cores.

Meaning of symbols:

Main Content: To start with we would establish the result for all even integers followed by all primes of the form 4n-1.

Note: that it is sufficient to establish the result for the integer '2' and even can be found by just substituting 2x, 2y, 2z in the original equation.

For e.g.

$1+ 4/2=1/x+1/y+1/z$

$\Rightarrow \quad 4/2n=2/x.n+1/y.n+1/z.n$

111r by $1+4/p=1/x+1/y+1/z$

=> $4/n.p=1/n.x+1/n.y+1/n.z$

Case I : $n=2$

We have $4/2=2=1/2+1/2+1/1$

=> $x=2,y=2,z=1$ give us ‘2’.

=> $x=2n$

$y=2n$

$z=n$

Is the general solution for all even integers.

=> $4/2n=1/2n+1/2n+1/n$

Above result holds for all even w.s.

Case II: All odd w.s of the form $4n-1$

(a) $n=2n$=even.

Let $4.2n-1=p$

$\Rightarrow$ $1/4n+1/4n+1/2pn=4/p$ gives a general solution.

It may be easily checked as follows.

LHS = $1/2n+1/2pn=p+1/2np=4.2n-1-1/2np=4/p$

(b) $n=2n+1$=odd

Let $4(2n+1) – 1=p$

The general solution would be

$1/2(2n+1)+1/2(2n+1)+1/p(2n+1)=4/p$

Since

LHS = 1/(2n+1)+1/p(2n+1)=4/p

=p+1/(2n+1).p =4.(2n+1)/(2n+1).p =4/p

⇨ All odd w.s of the form

⇨ 4n-1 are covered in the cases (a) & (b).

Case III: Now comes the must controversial case of 4.n+1.

(a) When n=2n+1=odd their does exists a general solution

i.e. 1/2p(n+1) +1/2(n+1) +1/p(n+1) =4/p

=> p+1/2(n+1).p = 1/p(n+1)

= p+3/2(n+1).p = 4.(2n+1)+1+z/2(n+1).p

(b) Now only left out cases is 4.2n+1 or P=8n+1

Dividing n as 3n, 3n+1 & 3n-1 we can get better results.

For n= 3n+1; P=23n+9 =3 [8n+3]

Which is resolved in the case of 4n-1=3 and its multiples.

Further case of n=3n-1 can be resolved as follows.

1/6n + 1/6np + 1/n.p =4/p

Since LHS = P+1/6np +1/np

= P+7/6np

=8(3n-1)+1+7/6np

=24n/6np=4/p= R.H.S.

The final case rather undecidable case is all primes of the form 24n+1

There is some success in this case too.

For eg.

If $4/p = 1/m + 1/mkp + 1/mkp$

s.t. $(4m-1). K-1 = P$

Then above eqn. Is valid.

=> Above condition holds it

$(4m-1) . K-1 = P$

or $P+1 = (4m-1). K$

or $24n+2 = 2(12n+1) = (4m-1). K$

=> $12n+1$ = composite of the form

$(4m_1-1)(4m_2-1). K_1$ etc.

Further one more possible case may occur

i.e.

$4/P = 1/m + 1/n + 1/e$

s.t. $(4m-p).n-m.p = 1$

Because it impales $e = 1/mnp$

We will conclude the article with one example of each of above cases.

For example 73=24.3+1= prime is our starting number for the check.

$4/73 = 1/21 + 1/2.\ 73 + 1/2.21.73$

Here 4021-73=11

In second step 11.2-21=1

=>   (4.21-73) .2-21=1 Which depicts our before mentioned case.

Also IInd prime of the form 24n+1 is 97.

We have

4/97 =1/28 =1/28.97 +1/14.97

We have

4.28-97 =112-97 =15

Also   15.32-28 =2

It seems that all primes of the form P=24n+1 does satisfy the above equation but no general method is easily traceable. Leaving it on that note it is left for the reader to accept the challenge.